Programs can be written in any language, and should be accompanied by a short description of how to compile and run them (you can put this in the comments at the head of the source code). Assignments will be submitted using git. All registered students should have received a document listing their username and password for bender.astro.sunysb.edu, along with some instructions on how to access their git repository.

1. (from Pang) Write a function that returns the minimum of a single-variable function, \( g(x) \) in a given region, \([a, b]\). Assume that the first-order and second-order derivatives of \( g(x) \) are not explicitly given. Test the routine with some well-known functions, e.g., \( g(x) = x^2 \).

2. ODEs (based on Pang) The equation of motion for a damped driven pendulum is:

\[
ml\ddot{\theta} = F_g + F_d + F_r
\]

where \( \theta \) is the angle of the pendulum from vertical, \( F_d = f_0 \cos \omega_d t \) is the driving force with amplitude \( f_0 \) and driving frequency \( \omega_d \), \( F_d = -\kappa \dot{\omega} \) is the resistive force and \( F_g = -mg \sin \theta \) is the gravitational force on the pendulum.

We can write this as a system of the form:

\[
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= -q\omega - \sin \theta + b \cos \omega_d t
\end{align*}
\]

where \( q \) is a scaled damping coefficient and \( b \) is a scaled forcing amplitude. We also took \( L = g \) to simplify things.

(a) Integrate this system using 4th-order Runge-Kutta—use a uniform timestep (later in the semester, we’ll reuse this data for a time-series analysis). Choose the parameters \( q = 0.5, b = 0.9, \) and \( \omega_d = 2/3 \). Integrate for several 100 periods. Pick a timestep that seems to give a converged solution.

Notice that if the driving force dominates, the pendulum can “flip over”, and \( \theta \) can fall outside of the range \([-\pi, \pi]\). But \( \omega \) is periodic with \( \theta \pm 2\pi \). To make a reasonable analysis, you need to correct \( \theta \) to fall within \([-\pi, \pi]\) by adding/subtracting multiples of \( 2\pi \).

Make a plot of the pendulum in the \( \omega - \theta \) plane. You should see a periodic motion.

(b) Slowly increase the parameter \( b \) by steps of 0.05 and, for each value of \( b \), integrate the system and make the \( \omega - \theta \) plot.

At which \( b \) does the system appear to become chaotic?

(You might want to google around for information on damped driven pendulums and chaos to better understand what you see.)

Please provide hard copies of your plots with the parameters labeled.