This is an open question. I want you to explore, in some detail, a numerical method of your choice that we didn’t cover in class (or go a lot deeper on something we just touched on). You should pick something that has an application that interests you.

You should hand in a commented example code and copies of any output (PDFs in your git repo are best), as well as a brief (1–2 pages) explanation of what the algorithm is doing and what your application is. The main goal is for you to think about some new numerical technique, everything else is pretty much open to you to explore.

Here are some potential projects to inspire you:

1. Investigate the Numerov method for Schrödinger’s equation—this is a special ODE integrator that is designed to work on second-order ODEs directly. Write a solver uses shooting to find bound states and eigenvalues for a general potential.

   Try some potentials with known solutions and some with unknown solutions and plot the wave function, etc. This is like we explored in class. For a new component, investigate some scattering solutions, \( E > V \).

2. Solve the eigenvalue Schrödinger problem using matrix methods. Here you will write the solution as a series of some basis functions and the linear algebra will give you approximations to the wavefunction and eigenvalues up to some cutoff, depending on how many terms you keep in your series. Newman discusses this a bit.

3. Solve the time-dependent Schrödinger equation. This is now a PDE. You can find a variety of methods online and in texts. Do some tunneling calculations with various potentials.

4. Investigate finite-element methods for PDEs and solve one or more of our model PDEs with these techniques.

5. Investigate spectral methods for PDEs and solve one or more of our model PDEs with these techniques.

6. We explored geometric multigrid, where the coarsening of the problem is done on the problem domain. Algebraic multigrid does the coarsening of the matrix itself. Explore algebraic multigrid and apply it to some of our problems.

7. Write a two-dimensional solver for Burgers’ equation and experiment with various different initial conditions.

8. Smoothed particle hydrodynamics is a method where particles map out the density of the flow and continuous quantities are constructed by integrating over the particle distribution (with some kernel function). Write a basic SPH solver (in 1-d) for advection and do some test problems.
9. There are a lot of matrix methods we skipped, like the conjugate gradient technique and singular-value decomposition. Explore one of these and compare to the techniques we covered in class.

10. The fast multipole method (FMM) or a tree-method can be applied to N-body calculations (like computing the gravitational force of a collection of particles). Explore this and construct a simple FMM or tree solver for a collection of particles.

11. For a bounded charge or mass distribution, a multipole expansion can be used to find the potential both inside and outside of the source by expanding in spherical harmonics. These expansions are often used to find the potential on the boundary of a domain to represent an isolated system. The Jackson E&M text is a classic reference for multipole expansions. Code up a method for doing a multipole expansion and test it with some distributions with known potentials.

12. MCMC can also be used for parameter fitting for scientific data. There are some references to this on the course webpage. Code up some examples of this and describe how it works.