1. (this is Newman 10.11) **Dimer covering problem**. Dimers are two-atom polymers. Imagine dimers landing on the surface of a solid, falling into the gaps between other dimers. They will cover the surface looking something like this:

![Figure 1: A 10×10 lattice with dimers.](image)

No dimers are allowed to overlap. Here we will use simulated annealing to find out how many dimers can fit in a \( L \times L \) square (we know the answer is simply \( L^2/2 \), but there are more complicated versions of this problem where we might not know the answer ahead of time).

Write a program to solve this problem using simulated annealing on a 50×50 lattice. The energy of the system is *minus* the number of dimers, so that is minimized when the dimers are a maximum. The moves for the Markov chain are:

- Choose two adjacent sites on the lattice at random
- If those two sites are currently occupied by a single dimer, remove the dimer from the lattice
- If, instead, they are currently both empty, add a dimer
• Otherwise, do nothing

Use an exponential cooling schedule and try out different time constants. A reasonable first value to try is $\tau = 10000$ steps. For faster cooling schedules, you should see that the solutions found are poorer—a smaller fraction of the lattice is filled with dimers and there are larger holes between them. For slower schedules, the calculation can find quite good, but usually not perfect, coverings of the lattice.