1. Faraday cage. The electric potential satisfies a Poisson equation:

\[ \nabla^2 \phi = \frac{\rho_c}{\epsilon_0} \]  

(1)

where \( \rho_c \) is the charge density and \( \epsilon_0 \) is the permittivity of vacuum.

Consider a simple Faraday cage (we are representing this as a 2-d plane). We have a unit-square box with 8 grounded points (you can think of these as wires running out of the page). The configuration appears as follows:

The potential on the 4 sides of the domain are as shown in the figure. The interior points (wires) are
at:

\[
\begin{pmatrix}
1 & 2 \\
3' & 3 \\
\end{pmatrix}
\begin{pmatrix}
1 & 2 \\
2' & 3 \\
\end{pmatrix}
\begin{pmatrix}
2 & 2 \\
3' & 3 \\
\end{pmatrix}
\begin{pmatrix}
1 & 1 \\
3' & 2 \\
\end{pmatrix}
\begin{pmatrix}
2 & 1 \\
3' & 2 \\
\end{pmatrix}
\begin{pmatrix}
1 & 1 \\
3' & 3 \\
\end{pmatrix}
\begin{pmatrix}
1 & 1 \\
2' & 3 \\
\end{pmatrix}
\begin{pmatrix}
2 & 1 \\
3' & 3 \\
\end{pmatrix}
\]

Since there are no charges, we are just solving \( \nabla^2 \phi = 0 \) and the plates act as additional boundaries.

You can use either a node-centered or cell-centered grid. Solve for the potential in the domain using smoothing/relaxation. For either choice of grid, implement the wires as simply the values of the potential at the points in the grid matching the geometry defined above. These points should not be updated during the relaxation—just keep them fixed at \( \phi = 0 \).

To check for convergence, check the norm of the residual, and continue smoothing until the error is less than \( 10^{-6} \). Note: do not compute a residual in the zones occupied by the plates.

Plot the potential, and optionally the electric field, \( E = -\nabla \phi \).

2. Time-dependent diffusion (from Newman). Consider diffusion through the Earth’s crust—this has a time-dependent boundary condition, the solar heating throughout the seasons. We’ll model this in 1-d.

The temperature at the surface of the Earth is:

\[
T_0(t) = A + B \sin \left( \frac{2\pi t}{\tau} \right)
\]

where \( \tau = 365 \) days, \( A = 10^\circ \) C is the average temperature and \( B = 12^\circ \) C captures the seasonal variation.

At a depth of 20 m, the temperature is relatively constant, which we take as \( 11^\circ \) C. This temperature is set by the heat flux coming from the interior of the Earth.

Take the thermal diffusivity of the Earth’s crust as \( D = 0.1 \) m\(^2\) day\(^{-1}\).

Solve the diffusion equation on a domain from the surface to a depth of 20 m. For the initial conditions, take \( T = 10^\circ \) C in all the interior zones. Evolve for 10 years—after a few years the initial guess at the temperature will have relaxed out.

You’ll need to pick a discretization, number of zones, and a timestep—use any of the methods we discussed in class.

For the final year, plot \( T(x) \) at 4 different times (corresponding to the 4 seasons). This will illustrate the seasonal variation of temperature with depth.